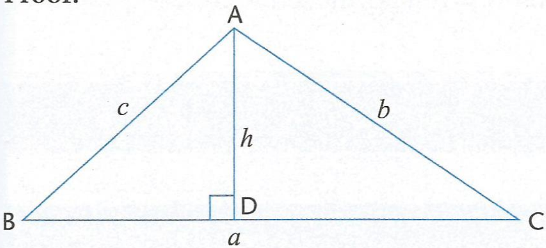
The sine rule and its proof

In any $\triangle ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Proof:



Acute-angled triangle

Let AD = h

= height of $\triangle ABC$ with base BC

$\sin B = \frac{h}{c}$ and $\sin C = \frac{h}{b}$

$\therefore h = c\sin B$ and $h = b\sin C$

Equate h on both sides:

$\therefore c\sin B = b\sin C$

Divide both sides by bc

$\frac{c \sin B}{bc} = \frac{b \sin C}{bc}$

$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$

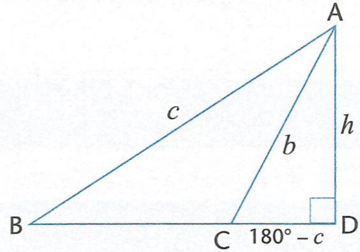
Let CE = h

= height of $\triangle ABC$ with base AB

Repeat the steps above to get:

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



Obtuse-angled triangle

Let AD = h

= height of $\triangle ABC$ with base BC

$\sin B = \frac{h}{c}$ and $\sin (180° - C) = \frac{h}{b}$

but $\sin C = \sin (180° - C)$

$\therefore h = c\sin B$ and $h = b\sin C$

Equate h on both sides:

$\therefore c\sin B = b\sin C$

Divide both sides by bc

$\frac{c \sin B}{bc} = \frac{b \sin C}{bc}$

$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$

Let AD = h

= height of $\triangle ABC$ with base AB

Repeat the steps above to get:

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ which is the same as

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$