Grade 12 Maths June 2017 Paper 2 Memo.

Question 1.

1.1. $r^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\therefore r = 13$ ✓

$\cos\alpha = \frac{12}{13}$ ✓

1.2. $T\hat{O}R + 90° + \alpha = 180°$ ✓

$\therefore T\hat{O}R = 90° - \alpha$ ✓

1.3. $\cos(90° - \alpha) = \frac{7{,}5}{OT}$ ✓

$\therefore OT = 7{,}5 \times \sin\alpha$ ✓

$= 7{,}5 \times \frac{5}{13}$ ✓

$= \frac{75}{26}$ ✓

1.4. $LHS = \frac{\sin(90° + x) \cdot \cos x \cdot \tan(-x)}{\cos(180° - x)}$

$= \frac{\cos x \cdot \cos x \cdot (-\sin x)}{-\cos x \cdot \cos x}$ ✓ ✓ ✓

$= \sin x$ ✓

$= RHS$

Question 3

3.1) $\sin 68^\circ . \cos 22^\circ + \cos 68^\circ . \sin 22^\circ$

$= \sin(68^\circ + 22^\circ)$

$= \sin 90^\circ$

$= 1$

3.2) $\cos(90^\circ + \theta) = \cos 90^\circ \overset{\cos}{\cancel{\sin}}\theta - \sin 90^\circ . \overset{\sin}{\cancel{\cos}}\theta$

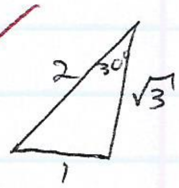
$= 0 - (1) . \overset{\sin}{\cancel{\cos}}\theta$

$= -\sin\theta$

3.3) $LHS = \cos(30^\circ + x)$

$= \cos 30^\circ . \cos x - \sin 30^\circ . \sin x$

$= \frac{\sqrt{3}}{2} . \cos x - \frac{1}{2} . \sin x$



$= \frac{\sqrt{3}\cos x - \sin x}{2} = RHS$

3.4) $2\sin 30^\circ . \cos 30^\circ = \sin 2(30^\circ)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

Question 4.

4.1) $AC^2 = 12^2 + 20^2 - 2(12)(20)\cos 110°$

$= 708,169 \ldots$

$\therefore AC = 26,6$

4.2) $\dfrac{\sin B\hat{A}C}{12} = \dfrac{\sin B}{26,6}$

$\therefore \sin B\hat{A}C = \dfrac{12 \times \sin 110°}{26,6}$

$= 0,423 \ldots$

$\therefore B\hat{A}C = 25°$

4.3) $AC^2 = 7^2 + 28^2 - 2(7)(28)\cos D$

$\dfrac{26,6^2 - 7^2 - 28^2}{-(7)(28)} = \cos D$

$\dfrac{16}{25}$ or $0,64 = \cos D$

$\therefore D = 50°$

4.4) $ABCD = \Delta ABC + \Delta ADC$

$= \frac{1}{2}(12)(20)\sin 110° + \frac{1}{2}(7)(28)\sin(D \text{ or } 50°)$

$= 112,763 \ldots + 75,072 \ldots$

$= 187,8$

QUESTION/VRAAG ~~4~~ 2

~~1.1~~ 2.1	$y=-3x+k$ $-3=(-3)(-1)+k$ **OR** By inspection, using the gradient: $k=-6$ $k=-6$	✓ substitution of $(-1\,;-3)$ ✓ $k=-6$ (2)
~~1.2~~ 2.2	$\frac{x_A-x_B}{2}=x_P$ $\frac{y_A+y_B}{2}=y_P$ $\frac{-1-x_B}{2}=\frac{5}{2}$ **and** $\frac{-3+y_B}{2}=1$ **OR** By using translation: $B(6\,;5)$ $x_B=6$ $y_B=5$ $\therefore B(6\,;5)$	✓ 6 ✓ 5 (2)
~~1.3~~ 2.3	$m_{AB}=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-3)}{6-(-1)}=\frac{8}{7}$ **OR** $m_{AB}=\frac{y_2-y_1}{x_2-x_1}=\frac{1-(-3)}{2.5-(-1)}=\frac{8}{7}$	✓ substitution ✓ gradient (2)

~~1.4~~ 2.4	$\tan\beta=m_{AD}=-3$ $\beta=108{,}43°$ $\tan\alpha=m_{AB}=\frac{8}{7}$ $\alpha=48.81°$ $\theta=108.43°-48.81°$ $\theta=59.62°$ **OR** $\tan\beta=m_{AD}=-3$ $\beta=108.43°$ $CDO=18.43°$ $\tan\alpha=m_{AB}=\frac{8}{7}$ $\alpha=48.81°$ $\theta=18.43°+(90°-48.81°)$ $\theta=59.62°$	✓ $\tan\beta=-3$ ✓ $\beta=108.43°$ ✓ $\tan\alpha=\frac{8}{7}$ ✓ $\alpha=48.81°$ ✓ $\theta=59.62°$ (5) ✓ $\tan\beta=-3$ ✓ $\beta=108.43°$ ✓ $\tan\alpha=\frac{8}{7}$ ✓ $\alpha=48.81°$ ✓ $\theta=59.62°$ (5)

Q2

2.5	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0+1)^2 + (-6+3)^2}$ $= \sqrt{10}$	✓ substitution into distance formula ✓ $\sqrt{10}$ (2)
2.6	$AC = 2\,AD$ $= 2\sqrt{10}$ $CB^2 = AC^2 + AB^2 - 2AC.AB.\cos\theta$ $= (2\sqrt{10})^2 + (\sqrt{113})^2 - 2(2\sqrt{10})(\sqrt{113})\cos 59.62°$ $= 84.998...$ $CB = 9.22$ units **OR** $D(0\,;-6)$, $A(-1\,;-3)$, $AC = 2AD$ So $x_C - x_A = 2(x_A - x_D)$: $x_C - 1 = 2(-1-0)$, $x_C = -3$ $y_C - y_A = 2(y_A - y_D)$ $y_C - 3 = 2(-3-6)$, $y_C = 3$ The coordinates of C are $(-3\,;3)$. $CB = \sqrt{(6-(-3))^2 + (5-3)^2}$ $= 9.22$ units	✓ $AC = 2\sqrt{10}$ ✓ using cosine rule ✓ substitution ✓ 84.998... ✓ 9.22 (5) ✓✓✓ $C(-3\,;3)$ ✓ substitution into distance formula ✓ 9.22 (5) **[18]**

QUESTION ~~7~~ 5

5.1	$A\hat{B}D = \theta$ [alternate ∠s, ∥ lines] $\cos\theta = \frac{BD}{AB} = \frac{64}{81}$ $\theta = 38°$ OR *OF* $\sin B\hat{A}D = \frac{64}{81}$ $B\hat{A}D = 52.18°$ $\theta = 38°$	✓ correct trig ratio ✓ substitution into correct ratio ✓ answer (to the nearest degree) (3) ✓ correct trig ratio ✓ substitution into correct ratio ✓ answer (to the nearest degree) (3)
5.2	$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ $= 81^2 + 87^2 - 2(81)(87)\cos 82.6°$ $= 12314.754...$ $BC = 110.97$ m	✓ use cosine rule ✓ correct substitution into cosine rule ✓ answer (3)

Q5

5.3	$\frac{\sin D\hat{C}B}{BD} = \frac{\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{BD \sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{64 \sin 110°}{110.97}$ $\therefore D\hat{C}B = 32.82°$	✓ use sine rule ✓ substitution ✓ answer (3) [9]

QUESTION 6

6.1	$\hat{A}_2 = x$ (∠s in same seg) $\hat{D}_2 = x$ (∠s opp = sides) $\hat{E}_2 = x$ (= chs = ∠s) or (∠s in same seg) $\hat{A}_3 = x$ (tan-chord theorem)	✓ $\hat{A}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ reason ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{A}_3 = x$ ✓ reason (8)
6.2	In Δ ABE and ΔDFE 1. $\hat{E}_2 = \hat{E}_1$ $(= x)$ 2. $\hat{D}_3 = 90°$ (∠s in semicircle) $B\hat{A}E = 90°$ (tan ⊥ rad) $B\hat{A}E = \hat{D}_3$ Δ ABE $\|\|\|$ ΔDFE (∠∠∠) $\frac{BE}{FE} = \frac{AE}{DE}$ ($\|\|\|$ Δs) BE.DE = AE.FE	✓ $\hat{E}_2 = \hat{E}_1$ ✓ $\hat{D}_3 = 90°$ ✓ reason ✓ $B\hat{A}E = 90°$ ✓ reason ✓ $\frac{BE}{FE} = \frac{AE}{DE}$ ✓ $\|\|\|$ Δs (7)
6.3	$\hat{D}_1 = 90° - x$ (∠s on str line) $\hat{B}_1 = 90° - x$ (∠ sum Δ) $\hat{B}_1 = \hat{D}_1$	✓ $\hat{D}_1 = 90° - x$ ✓ reason ✓ $\hat{B}_1 = 90° - x$ ✓ reason (4) [19]

Question 7.

7.1.1 $f(x) = 3 - 2\sin^2 x$

$-1 \leq \sin x \leq 1$

$0 \leq \sin^2 x \leq 1$

$0 \geq -2\sin^2 x \geq -2$ (× −2 , op change)

$\therefore -2 \leq -2\sin^2 x \leq 0$

$\therefore 1 \leq 3 - 2\sin^2 x \leq 3$ (+ 3)

$\therefore 1 \leq y \leq 3$

or $y \in [1;3]$

7.1.2. f has min when $\sin^2 x = 1$

$\therefore \sin x = \pm 1$

$\therefore x = -90°$ or $x = 90°$

7.2) $LHS = 1 - \cos 2Q$

$= 1 - (1 - 2\sin^2 Q)$

$= 2\sin^2 Q$

7.3) a) $R = 180° - (P + Q)$

$\sin 2R = \cancel{\sin 2(} \sin 2(180° - (P+Q))$

$= \sin(360° - 2(P+Q))$

$= \sin(360° - (2P + 2Q))$

$= -\sin(2P + 2Q)$

[3]

7.3) b) $LHS = \sin 2P + \sin 2Q + \sin 2R$

$= \sin 2P + \sin 2Q - \sin(2P + 2Q)$

$= \sin 2P + \sin 2Q - [\sin 2P \cos 2Q + \cos 2P \sin 2Q]$

$= \sin 2P + \sin 2Q - \sin 2P \cos 2Q - \cos 2P \sin 2Q$

$= \sin 2P(1 - \cos 2Q) + \sin 2Q(1 - \cos 2P)$

$= \sin 2P(2\sin^2 Q) + \sin 2Q(2\sin^2 P)$

$= 2\sin P \cos P . 2\sin^2 Q + 2\sin Q \cos Q . 2\sin^2 P$

$= 4\sin P \cos P \sin^2 Q + 4\sin Q \cos Q . \sin^2 P$

$= 4\sin P \sin Q(\sin Q \cos P + \sin P \cos Q)$

$= 4\sin P \sin Q(\sin(Q + P))$

$= 4\sin P \sin Q(\sin(180° - (Q + P)))$

$= 4\sin P \sin Q . \sin R$

QUESTION ~~7~~ 6

6.1	$\hat{A}_2 = x$ (∠s in same seg) $\hat{D}_2 = x$ (∠s opp = sides) $\hat{E}_2 = x$ (= chs = ∠s) or (∠s in same seg) $\hat{A}_3 = x$ (tan-chord theorem)	✓ $\hat{A}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ reason ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{A}_3 = x$ ✓ reason (8)
6.2	In Δ ABE and ΔDFE 1. $\hat{E}_2 = \hat{E}_1$ $(= x)$ 2. $\hat{D}_3 = 90°$ (∠s in semicircle) $B\hat{A}E = 90°$ (tan ⊥ rad) $B\hat{A}E = \hat{D}_3$ Δ ABE ||| ΔDFE (∠∠∠) $\frac{BE}{FE} = \frac{AE}{DE}$ (||| Δs) BE.DE = AE.FE	✓ $\hat{E}_2 = \hat{E}_1$ ✓ $\hat{D}_3 = 90°$ ✓ reason ✓ $B\hat{A}E = 90°$ ✓ reason ✓ $\frac{BE}{FE} = \frac{AE}{DE}$ ✓ ||| Δs (7)
6.3	$\hat{D}_1 = 90° - x$ (∠s on str line) $\hat{B}_1 = 90° - x$ (∠ sum Δ) $\hat{B}_1 = \hat{D}_1$	✓ $\hat{D}_1 = 90° - x$ ✓ reason ✓ $\hat{B}_1 = 90° - x$ ✓ reason (4) [19]

QUESTION 8

.1	the interior opposite angle *die teenoorstaande binnehoek*	✓ answer *antw* (1)

8.2

$\hat{V}_1 + \hat{V}_2 = 90°$	**∠ in semi-circle**/*∠ in halfsirkel*	✓ S ✓ R
$\hat{T}_2 = 90° - x$	**Tangent ⊥ diameter/radius**/*raaklyn ⊥ middellyn/radius*	✓ R
$\therefore \hat{C} = x$	Sum of the angles of triangle *Som van die hoeke van 'n driehoek*	✓ S
$\therefore \hat{S} = x$	**∠'s same segment**/*∠e in dieselfde segment*	✓ R
$\therefore V\hat{T}R = \hat{S}$		(5)

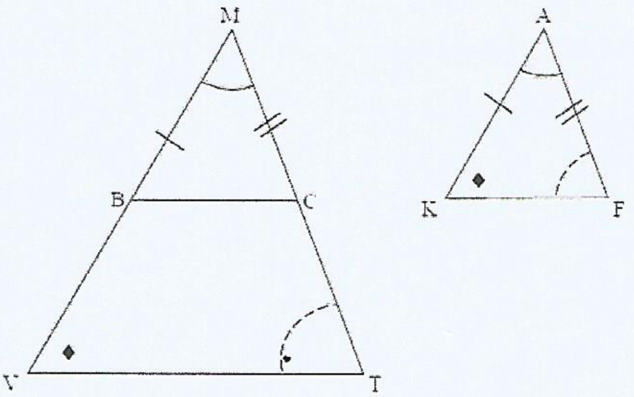
Q8

8.3

8.1	Equal chords subtend equal ∠s *Gelyke koorde onderspan gelyke ∠e*	✓ R (1)
8.2	$\hat{W}_4 = 30°$ (tan chord theorem *rkl-koordst*) $\hat{W}_1 = 30°$	✓ answer *antw* ✓ reason *rede* ✓ answer *antw* (3)
8.3(a)	$\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem *rkl-koordst*) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext ∠ of Δ *buite ∠ v Δ*) $\therefore \hat{S}_2 = 80°$ **OR/*OF*** $\hat{R}_2 = \hat{R}_3 = 30°$ (= chords subtend = ∠s = *kde onderspan* = ∠*e*) $\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem *rkl-koordst*) $\therefore \hat{S}_2 = 80°$	✓ S ✓R ✓ S (3) ✓ S ✓R ✓ S (3)

8.3(b)	$\hat{T}_2 = \hat{S}_2 = 80°$ (ext ∠ of cyclic quad *buite ∠ van koordevh*) $\hat{V} + \hat{W}_4 = \hat{T}_2$ (ext ∠ of Δ *buite ∠ van Δ*) $\therefore \hat{V} = 50°$	✓ S ✓ R ✓ S ✓ S (4)
8.4	In ΔRVW and *en* ΔRWS: $\hat{R}_2 = \hat{R}_3 = 30°$ (proven *bewys* in 9.3.1) $\hat{V} = \hat{W}_2 = 50°$ (proven *bewys* in 9.3.3) $V\hat{W}R = \hat{S}_2$ (3rd ∠ in Δ) $\therefore \Delta RVW \parallel\!\mid \Delta RWS$ (∠∠∠) $\therefore \frac{WR}{RV} = \frac{RS}{WR}$ (ΔRVW ΔRWS) $\therefore WR^2 = RV.RS$	✓ using the correct Δs *gebruik korrekte Δe* ✓ S ✓ S ✓ R (3rd ∠ in Δ) or (∠∠∠) ✓ S (5) [22]

QUESTION 9



9.1	Constr *Konstr* :	
	Draw line BC such that MB = AK and MC = AF	✓ constr *konstr*
	Trek lyn BC sodat MB = AK en MC = AF	
	Proof *Bewys* :	
	In ΔBMC and *en* ΔKAF	
	MB = AK [constr *konstr*]	
	$\hat{M} = \hat{A}$ [given *gegee*]	
	MC = AF [constr *konstr*]	
	$\Delta BMC \equiv \Delta KAF$ [s∠s]	✓ S R
	$\therefore M\hat{B}C = A\hat{K}F$ or $M\hat{C}B = A\hat{F}K$ [$\equiv \Delta$]	✓ S
	but *maar* $\hat{V} = \hat{K}$ or $\hat{T} = \hat{F}$ [given *gegee*]	
	$\therefore M\hat{B}C = \hat{V}$ or $M\hat{C}B = \hat{T}$	✓ S
	But these are corresponding ∠s *maar hulle is ooreenk ∠e*	
	$\therefore BC \parallel VT$ [corr ∠s = *ooreenk∠e* =]	✓ S R
	$\therefore \frac{MV}{MB} = \frac{MT}{MC}$ [prop theorem *eweredighst*: $BC \parallel VT$]	✓S ✓R
	but *maar* MB = AK and MC = AF [constr *konstr*]	
	$\therefore \frac{MV}{AK} = \frac{MT}{AF}$	(7)

Q9

9.2.1(a)	In ΔKGH and ΔKEF $\hat{K}$ is common *gemeen* $\hat{H}_2 = \hat{F}$ [ext $\angle$ cyclic quad *buite $\angle$ koordevh*] $\hat{G}_3 = \hat{E}$ [sum$\angle$s Δ OR ext $\angle$ cyclic quad *som$\angle$eΔ OR buite$\angle$koordevh*] $\therefore \Delta KGH \parallel\!\mid \Delta KEF$ [$\angle\angle\angle$]	✓ S ✓S ✓R ✓ naming third angle OR $\angle\angle\angle$ (4)
9.2.1(b)	$\frac{EF}{GH} = \frac{KE}{KG}$ [$\parallel\!\mid \Delta$s] $\therefore \frac{EF}{GH} = \frac{KE}{EF}$ [KG = EF] $\therefore EF^2 = KE.GH$	✓ S ✓ S (2)
9.2.1(c)	$\frac{KG}{KF} = \frac{EM}{EF}$ [prop theorem *eweredighst*: MG ∥ EK] but EF = KG [given *gegee*] $\frac{KG}{KF} = \frac{EM}{KG}$ $KG^2 = EM.KF$	✓ S ✓ R ✓ S (3)
9.2.2	KE.GH = EM.KF $EM = \frac{20 \times 4}{16}$ $= 5$ units	✓ KE.GH = EM.KF ✓ substitution ✓ answer (3) [19]