10 May 2017 Grade 12 Cycle test

Cornerstone College
MODERATED BY HOD
DATE: 02|05|2017
SIGNATURE:

Question 1

$$LHS = \frac{1+\sin 2A}{\cos 2A} = \frac{1 + 2\sin A \cdot \cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A + 2\sin A\cos A + \cos^2 A}{\cos^2 A - \sin^2 A}$$

$$= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A - \sin A)(\cos A + \sin A)}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A} = RK$$

(6)

Question 2

$$\sin x \cdot \cos x = \frac{2\sin x \cdot \cos x}{2}$$

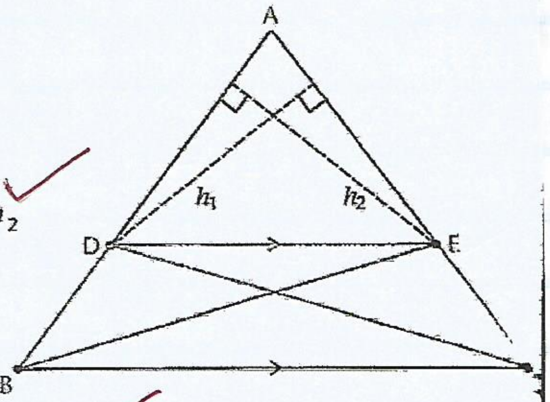
$$= \frac{\sin 2x}{2}$$

(2)

Question 3

Theorem: A line drawn parallel to one side of a triangle divides the other two sides proportionally.

Given: $\triangle ABC$ with $DE \parallel BC$, such that D lies on AB and E lies on AC.



Proof:

$$\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DBE} = \frac{\frac{1}{2}AD \times h_2}{\frac{1}{2}DB \times h_2} = \frac{AD}{DB}$$

| Common vertex E, same height h_2

$$\frac{\text{Area } \triangle AED}{\text{Area } \triangle ECD} = \frac{\frac{1}{2}AE \times h_1}{\frac{1}{2}EC \times h_1} = \frac{AE}{EC}$$

| Common vertex D, same height h_1

Area $\triangle DBE$ = Area $\triangle ECD$

| Common base DE, same height, $DE \parallel BC$

$$\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DBE} = \frac{\text{Area } \triangle AED}{\text{Area } \triangle ECD}$$ | Area $\triangle ADE$ is common and Area $\triangle DBE$ = Area $\triangle EC$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

(8)

Question 4

4.1. $\frac{AS}{SP} = \frac{AR}{RB}$ (Proportionality theorem, $RS \parallel BP$) ✓

$AR : AB = 3:5$ (Given)

$\therefore AR : RB = 3:2$ ✓

$\therefore \frac{AS}{SP} = \frac{3}{2} \implies AS:SP = 3:2$ ✓

(3)

4.2. $\frac{AS}{SP} = \frac{3}{2}$ (already proven)

But $\cancel{S}\overset{A}{}P = PC = 5x$ (Given) ✓

$\therefore \frac{AS}{SP+PC} = \frac{3x}{2x+5x}$ ✓

$= \frac{3}{7} \implies AS:SP = 3:7$ ✓

(3)

4.3 $\frac{RT}{TC} = \frac{SP}{PC}$ ✓ (Proportionality theorem, $RS \parallel TP$)

$= \frac{2}{5} \implies RT:TC = 2:5$ ✓

(2)

4.4. $\frac{\text{area } \Delta\, TPC}{\text{area } \Delta\, RSC} = \frac{\frac{1}{2}(PC)(TC)\sin \hat{C}_1}{\frac{1}{2}(SC)(RC)\sin \hat{C}_1}$ ✓ (Sin area rule) ✓

$= \frac{(5x)\cdot(5p)}{(7x)\cdot(7p)}$ ✓ ✓

$= \frac{25}{49}$ ✓

(5)

Question 5

In ΔCBA and $\Delta \cancel{B}CDB$ is:

$\Delta CBA \,|||\, \Delta C$

Question 5

$OB \perp BD$ (Radius $\perp$ tangent)

$A\hat{C}B = 90°$ ($\angle$ on diameter)

$\therefore \triangle CBA \,|||\, \triangle BDA$ (In $\triangle ABD$, $A\hat{B}D = 90°$, $BC \perp AD$)

$\therefore \dfrac{BA}{DA} = \dfrac{CA}{BA}$ ($\triangle CBA \,|||\, \triangle BDA$)

$\therefore AB^2 = AC \cdot AD$ (5)

Question 6.

6.1. $\dfrac{BC}{2} = \sin\theta$

$\therefore \underline{BC = 2\sin\theta}$ (2)

6.2. $\dfrac{BD}{\sin(90°-\theta)} = \dfrac{BC}{\sin 2\theta}$

$= \dfrac{2\sin\theta}{\sin 2\theta}$

$\therefore BD = \dfrac{2\sin\theta \cdot \sin(90°-\theta)}{\sin 2\theta}$

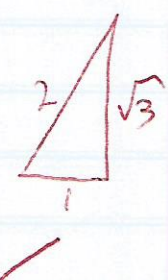
$= \dfrac{2\sin\theta \cdot \cos\theta}{2\sin\theta\cos\theta} = 1$ (5)

6.3. In $\triangle ABD$ is:

$AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos A\hat{B}D$

$\cos A\hat{B}D = \dfrac{AD^2 - AB^2 - BD^2}{-2(AB)(BD)}$

$= \dfrac{\sqrt{3}^2 - 2^2 - 1^2}{-2\cancel{(\sqrt{3})}(2)(1)}$



$= \dfrac{3-4-1}{-4}$

$= \dfrac{1}{2} \Rightarrow A\hat{B}D = \cancel{30°}\ 60°$ (4)

Question 7.

If $(x+1)$ is a factor of $f(x)$

then $f(-1) = 0$

$\therefore f(-1) = (-1)^3 + a(-1)^2 + b(-1) - 4 = 0$

$\therefore a - b = 5 \quad \ldots\ldots (1)$

If $(x-2)$ is a factor of $f(x)$

then $f(+2) = 0$

$\therefore \quad f(+2) = \overset{8}{(+2)^3} + \overset{4}{a(+2)^2} + b(+2) - 4 = 0$

$\therefore \quad 4a - 2b = 12$

$\therefore \quad 2a - b = 6 \quad \ldots (2)$

$(2) - (1): \quad a = 1$

$\therefore \quad b = -4$

(6)

$(x-2)$ is factor

$\Rightarrow f(2) = 0$

$\therefore \quad 2^3 + 4a + 2b - 4 = 0$

$4a + 2b + 4 = 0$

$2a + b = -2 \quad \ldots (2)$

$(1) + (2)$:

$3a = \cancel{-2}\,3$

$a = \cancel{-\tfrac{2}{3}}\,1 \quad 1$

Replace $a = \cancel{-\tfrac{2}{3}}\,1$ in (1)

$\therefore \; 1\cancel{-\tfrac{2}{3}} - b = 5$

$-b = \cancel{5\tfrac{2}{3}}\; -4$

$b = -\cancel{5\tfrac{2}{3}} \;/\; -\cancel{\tfrac{17}{3}}$