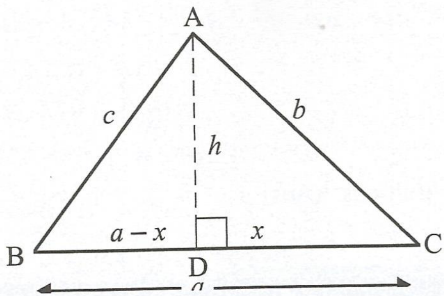
The cosine rule and its proof

For triangles that do not have a known side-angle pair you cannot use the sine rule. You therefore need to know and prove the cosine rule.

In any $\triangle ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$ or $b^2 = a^2 + c^2 - 2ac\cos B$ or $c^2 = a^2 + b^2 - 2ab\cos C$



Acute-angled triangle

Let AD = h

= height of $\triangle ABC$ with base BC

Let DC = x $\therefore$ BD = $a - x$

Apply Pythagoras' Theorem to $\triangle ABD$ to find c^2

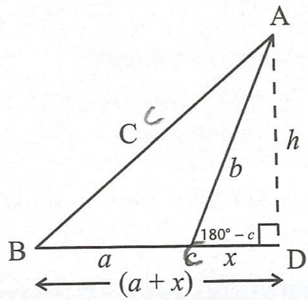
$$\begin{aligned} c^2 &= h^2 + (a - x)^2 \\ &= h^2 + a^2 - 2ax + x^2 \\ &= a^2 + (h^2 + x^2) - 2ax \\ &= a^2 + b^2 - 2ax \end{aligned}$$

$b^2 = h^2 + x^2$ | Pythagoras in $\triangle ADC$

$\frac{x}{b} = \cos C$

$\therefore x = b\cos C$

$\therefore c^2 = a^2 + b^2 - 2ab\cos C$



Obtuse-angled triangle

Let AD = h

= height of $\triangle ABC$ with base BC

Let DC = x $\therefore$ BD = $a + x$

Apply Pythagoras' Theorem to $\triangle ABD$ to find c^2

$$\begin{aligned} c^2 &= h^2 + (a + x)^2 \\ &= h^2 + a^2 + 2ax + x^2 \\ &= a^2 + (h^2 + x^2) + 2ax \\ &= a^2 + b^2 + 2ax \end{aligned}$$

$b^2 = h^2 + x^2$ | Pythagorus in $\triangle ADC$

$\frac{x}{b} = \cos(180° - C)$

$\therefore x = -b\cos C$ | $\cos(180° - C) = -\cos C$

$\therefore c^2 = a^2 + b^2 - 2ab\cos C$